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# **The Off-peak Demand for Train Kilometres and Train Tickets**

## **A Microeconometric Analysis**

**Daniel Van Vuuren and Piet Rietveld**

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### **Abstract**

In this paper the authors model the demand for train kilometres with a disaggregate structural demand model, thereby recognising the fact that consumers have to make a simultaneous choice for this demand and for the type of ticket with which they want to travel. The model is in line with microeconomic theory, which implies that the choice of ticket type is closely linked to the indirect utility function. Special attention has been paid to the possibility that individuals do not buy the ticket that would have been most advantageous, given their actual demand for train kilometres. A result implies that the average traveller behaves as if the price of a reduction card were more expensive than it actually is, which not only has considerable implications for the railway company whose aim is to maximise its revenues, but also for the government whose aim is to stimulate the use of public transport.

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## Introduction

Most railway companies offer various types of train tickets. Apart from full tariff tickets, travellers are often able to opt for various kinds of tickets that involve a fixed fee in order to “participate” and a marginal price that is lower than the “full tariff” marginal price. This results in a declining price schedule: travellers with high demand for train kilometres face a lower marginal (and average) price than travellers with low demand. The theoretical justifications for such a differentiated price structure have by now become common knowledge among public economists in general and transport economists in particular; see Brown and Sibley (1986) and Carbajo (1988), respectively, for an overview.

This paper aims at developing and estimating a disaggregate model for the demand for train kilometres. This (individual) demand typically depends on price and hence on the choice of train ticket, which implies that the choice of ticket type is a typical explanatory variable for the demand for train kilometres. However, this choice of ticket type is a typical *endogenous variable*, as in turn this variable will depend on the demand for train kilometres. When setting up a demand model it is therefore important to recognise the close relation of the level of the demand for train kilometres on the one hand and the choice of ticket type on the other. Simply using the choice of ticket type as an explanatory variable for the demand for train kilometres is inconsistent as this latter variable will also affect the choice of ticket type. Conversely, it makes no sense to explain separately the observed choice of ticket type from the observed demand for train kilometres. Using more technical terms, such analyses lead to biased estimates due to neglecting the endogeneity of an explanatory variable. It is therefore important to specify a simultaneous model that both explains the demand for train kilometres from the choice of ticket type, in particular from the price of this ticket type, and also explains the choice of ticket type from the demand for train kilometres. Making use of a maximum likelihood procedure this simultaneous model can be consistently estimated. Regarding the model structure, there are many possibilities for such a simultaneous model, but from the point of view of an economist the microeconomic framework is most appealing due to its strong theoretical justifications. This implies that the choice of ticket type is closely linked to the indirect utility function, which can be retrieved from the demand function for train kilometres by making use of Roy’s identity.

This approach dates back to Burtless and Hausman (1978)’s seminal paper on the supply of labour by individuals. Since then many empirical

studies that involve *nonlinear budget constraints* have been carried out. Among others, one can find applications in the fields of *electricity demand* (Dubin and McFadden, 1984; Herriges and King, 1994), and the *demand for housing* (King, 1980; Hausman and Wise, 1980; Hoyt and Rosenthal, 1990). An important distinction between the current work and most of these studies concerns the nature of the discrete choice. In most applications, such as labour supply, the discrete choice is automatically tied to the continuous choice and therefore has an implicit nature. For example, a person who chooses to work  $H$  hours a week will automatically face the tax rate that goes with this amount  $H$ . In other applications however, such as the demand for telecommunication and travel demand, the discrete choice has an explicit nature. As the choice of the ticket type is *optional*, it is likely that people may in some cases make choices that are *a priori unexpected*. This can be due to optimisation error of the individual, which may for example be caused by inadequate information. On the other hand it should be realised that the choice of ticket type has to be made in advance — usually on a yearly basis — which may be the cause of risk-averse behaviour of the individual. This hypothesis is confirmed by the data, which suggest that people are conservative with regard to the purchase of a reduction card. This risk aversion, together with the existence of non-monetary application costs such as waiting time and effort to submit an application form, is sufficient reason to incorporate a threshold parameter into the model that measures to what extent people are reluctant to buy a reduced fare card.

In the context of transport modelling, the present analysis has links with the approach of Abdelwahib and Sargious (1992), who model the demand for freight transport with a joint model for mode choice and shipment size. Studies by de Jong (1990) and Hensher, Milthorpe and Smith (1990), making use of discrete/continuous models for car ownership and car use, are also linked to the approach in this paper. It is remarkable, however, that in general there has been scarce attention to a discrete/continuous analysis of the demand for transport. Most studies up to present have either made use of discrete choice models, neglecting the continuous choice, or aggregate demand models, often neglecting the endogeneity of certain explanatory variables, such as the price variable. The current study improves on earlier approaches as it provides a theoretically justified framework for the travel behaviour of individuals and it recognises the simultaneous nature of train kilometres and train tickets in a consistent way. More concretely, this study provides estimates of price elasticities for train kilometres during off-peak hours and monetarises the travellers' reluctance to buy reduced fare cards through the inclusion of a "take-up cost" parameter. This study limits itself to the demand for Sec-

ond Class tickets during off-peak hours, but the model can be readily generalised for the demand during peak hours or even the demand for other transport modes.

In Section 2 the theoretical demand model for the combined demand for train kilometres and train tickets is set out. The data are described in Section 3. In this section special attention is paid to the observed combinations of train tickets and train kilometres. Indeed, the data suggest that in general consumers are reluctant to buy a reduction card. The transition from the theoretical economic model in Section 2 to an estimable econometric model is discussed in Section 4. In particular three issues are of interest in this step: (i) proper specification of the travellers' reluctance to buy reduced fare cards; (ii) allowance for individually differentiated tastes; and (iii) the stochastic specification of the model. Section 5 reports the estimates, and in particular compares the elasticity estimates to earlier estimates and discusses the implications of a positive take-up cost. Section 6 concludes and suggests several extensions of the approach in this paper.

## The Demand for Train Tickets and Train Kilometres: Microeconomic Theory

Following elementary microeconomic theory, the demand of an individual for a certain economic good is a function of the price of that good and of the individual's income. A possible specification of such a demand function for the case of train kilometres is the *double-log* specification:

$$\ln k = \alpha \ln y + \beta \ln p + \delta. \quad (1)$$

In this equation the number of train kilometres travelled by an individual is denoted by  $k$ , income is denoted by  $y$ , the price of a train kilometre is denoted by  $p$ , and  $\delta$  is simply a constant. This demand equation fixes a two-good microeconomic framework, consisting of (a) train kilometres and (b) a composite commodity that represents consumption on all other goods. To stay in line with demand theory, and in particular with the homogeneity property of the demand function in income and price, one has to deflate both income and (kilometre) price by the price of this composite commodity. As usual in cross-section analyses it is assumed that the price of the composite commodity is equal to the "price" of income, implying a numeraire that equals 1. It is seen that in this specification both the income elasticity of demand and the price elasticity of demand are constant at  $\epsilon_y = \alpha$  and  $\epsilon_p = \beta$  respectively, which explains why this model is often labelled the *constant elasticity model*. The use of the

logarithmic transformation on the left-hand side has been strongly suggested by a preliminary analysis of our data, which showed that the frequency distribution of the demand for train kilometres is far better described by a lognormal distribution rather than a normal distribution. The use of logarithms on the right-hand side has the advantage that no direct restrictions are placed on the elasticity estimates, as is the case in many models, for example the CES and Cobb-Douglas utility specification and the linear expenditure system.<sup>1</sup>

Consider now the case that consumers can choose between different types of tickets, for example a full tariff ticket and a reduced tariff ticket where the consumers first have to buy a reduction card at fixed cost. This implies that consumers are able to choose their own marginal price of a train ticket. As is well known, straightforward least squares estimation of the model in (1) would yield biased estimates, because the demand for kilometres has a causal influence on the kilometre price.<sup>2</sup> Therefore, it is necessary to add an equation to the model that adequately describes the causality of kilometres on kilometre price. There are several ways to do this, but from an economic point of view it is certainly most appealing to let this additional equation be consistent with microeconomic theory. The proper way of doing so starts by deriving the indirect utility function from the demand equation in (1). This can be achieved by rewriting Roy's identity as a differential equation and making use of the implicit function theorem. Following Burtless and Hausman (1978), this procedure yields the following indirect utility function:

$$v(p, y) = \frac{1}{1 - \alpha} y^{1 - \alpha} - \frac{1}{1 + \beta} p^{1 + \beta} e^{\delta}. \quad (2)$$

Hence it is possible to determine the *conditional demand function* and the *conditional indirect utility function* for each type of train ticket. Suppose

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<sup>1</sup>See Hausman (1980) for a discussion. As an illustration, consider the semi-log specification  $\ln k = \theta_0 + \theta_1 y + \theta_2 p$ . The implied price elasticity of this model equals  $\theta_2 p$ , and hence it is seen that the higher the price one pays, the higher will be (the absolute value of) its implied price elasticity. This is particularly unlikely for the demand for train kilometres, because it is not expected that travellers with relatively low use are the ones with the highest price sensitivity. A relatively price sensitive traveller, however, would be tempted to explore the benefits of reduction cards, seasonal tickets, and so on. Resuming matters in a more general fashion, the constant elasticity model of (1) does not impose a similar restriction on the price elasticity, and has therefore been chosen to be the most suitable specification for this study.

<sup>2</sup>See Train (1986) for an intuitively appealing illustration of such endogeneity bias. Mannering (1986) shows, through an empirical comparison of (biased) least squares estimates with unbiased counterparts, that the inconsistent estimation of transport demand models indeed causes a substantial bias in estimates for price and income elasticities.

that one is able to choose between two ticket types:<sup>3</sup> First, one may simply travel with the standard *full fare* tickets and pay a marginal price of, say,  $p_f$ . The second possibility is to buy a reduced fare card at a fixed price of, say,  $p_r$ , and hence pay a marginal price that equals a fraction  $h$  of the full tariff price ( $0 \leq h < 1$ ). This gives a budget constraint similar to the one in Figure 1, where the budget frontier ABC is obtained as an optimal combination of the full tariff budget frontier ABE and the reduced tariff budget frontier DBC. Note that the length of AD is exactly the price of a reduction card  $p_r$ . To a utility maximising individual the consumption of a package on the curve DBE should be no option, as this is not in accordance with the conventional wisdom that utility is nondecreasing in income. (In later sections we will come back to this.) Consider now the case that the individual's optimal demand for train kilometres, given that he travels with full tariff tickets, locates on AB, and that his optimal demand for train kilometres, given that he travels at reduced tariff, locates at the segment BC. In the figure these conditional demand functions have been denoted by  $x_1^*$  and  $x_2^*$  respectively. The accompanying conditional indirect utilities  $v_1$  and  $v_2$  determine the depicted indifference curves " $u = v_1$ " and " $u = v_2$ ". It can now be seen that if:

- (i) the conditional indirect utility of travelling at reduced tariff exceeds the conditional indirect utility of travelling at full tariff:  $v_2 > v_1$ ; and
- (ii) utility is quasi-concave;

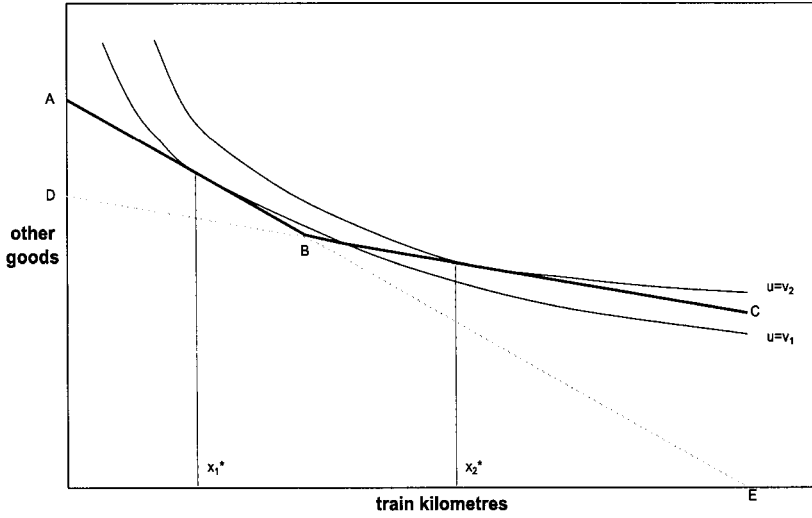
then the individual will prefer to buy a reduction card and simultaneously consume  $x_2^*$  kilometres at reduced tariff. The quasi-concavity condition guarantees that the indifference curves do not cross, so that demand at  $x_2^*$  is indeed optimal. Figure 2 depicts the case where the individual prefers to travel at full tariff, so the inequality in (i) reverses.<sup>4</sup> The case that the conditional demand for a reduced tariff card locates at DB can be ruled out, as the indirect utility that is inherent in such demand will always be inferior to the conditional demand for full tariff tickets that locates on AB — provided that the quasi-concavity condition holds.<sup>5</sup>

<sup>3</sup>The following can straightforwardly be generalised to the case where more ticket types are involved. See Pudney (1989) for more details.

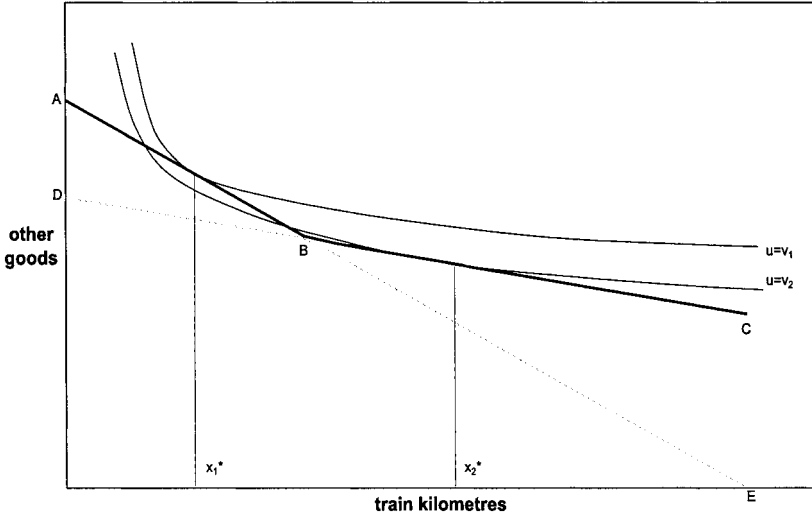
<sup>4</sup>Note that it is theoretically possible that one is indifferent between the two ticket types. In this case the relevant indifference curves coincide and go through both conditional demands  $x_1^*$  and  $x_2^*$ . In the empirical context this is usually neglected, as with the introduction of stochastic error terms into the model, the probability of being indifferent between the two ticket types equals zero and is therefore no longer relevant.

<sup>5</sup>As mentioned above, no consumer should be inclined to have demand on DBE. Hence, if  $x_2^*$  locates on DB, this implies that  $x_1^*$  must be at AB.

**Figure 1**  
Two ticket types: Preference of a reduction card ( $v_2 > v_1$ )



**Figure 2**  
Two ticket types: Preference of full tariff tickets ( $v_1 > v_2$ )



Returning to the model, the condition in (i) for preferring to travel at reduced tariff instead of full tariff rewrites as

$$v(hp_f, Y - p_r) > v(p_f, Y), \quad (3)$$



where total income is now denoted by  $Y$ . Having made the decision on which type of travel card one wants, the net income  $y$  equals  $Y - p_r$  for reduced tariff travellers and  $Y$  for full tariff travellers, respectively. Working out this equation with the expression for the indirect utility function in (2), we arrive at the following inequality, which will prove convenient as we will extend our model in the fourth section:

$$p_r < \gamma Y, \quad (4)$$

where

$$\gamma := 1 - \left\{ 1 - \frac{v(hp_f, 0) - v(p_f, 0)}{v(0, Y)} \right\}^{\frac{1}{1-\alpha}}. \quad (5)$$

It is seen that  $\gamma$  can be interpreted as the (maximum) fraction of income that is available for the purchase of a reduction card. Given his income  $Y$ , a traveller values the reduction card at  $\gamma Y$ , and hence decides to purchase a reduction card if, and only if, this valuation is higher than the actual price  $p_r$ , so that the purchase of this reduction card yields an implicit monetary benefit of  $\gamma Y - p_r$ . Note that  $\gamma$  is large for travellers who are relatively sensitive to price changes, while it is low for travellers who are relatively sensitive to income changes. Equations (4) and (5) therefore show that travellers who are relatively more sensitive to price changes than they are to income changes are more inclined to buy a reduced fare card than others.

Finally the quasi-concavity condition in (ii) writes as<sup>6</sup>

$$\beta < - \frac{\alpha p k}{y},$$

where  $y$  equals  $Y - p_r$  or  $Y$  respectively, dependent on whether one owns a reduction card or not, and  $p$  equals  $hp_f$  and  $p_f$  for the same respective cases. In less technical terms this inequality states that the compensated own-price-effect of train kilometres should be negative, a well known result in microeconomic theory.

## Data

In this paper we make use of a data set from the Netherlands Railways (NS), the so-called “Basisonderzoek” (BO). This data set was obtained

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<sup>6</sup>See Burtless and Hausman (1978) for details. Note that in the present two-good case, satisfaction of this inequality automatically dissolves the *integrability problem*, which demands symmetry and negative semidefiniteness of the Slutsky matrix.

through a survey during the period April 1992 to March 1993. The most important objective of the study was to analyse the profiles of travellers, in particular those of travellers by train. The survey consisted of two parts. First, a random sample from the population of Dutch households was drawn, after which one individual from the household was asked about personal characteristics such as age, gender and education. The subjects were also asked about their possession of train fare reduction tickets, car possession, and other factors of interest for the Netherlands Railways. Second, every subject was asked to keep a month's diary of his travel behaviour as far as railway transport was concerned. The subjects in the second data set are a subsample of those in the first, satisfying two selection criteria: (i) the subject indicated that he travelled by train during the past year; and (ii) the subject was willing to keep a travel diary. For the purpose of this paper we have linked both data sets in order to be able to include individual characteristics into the demand equation that was specified in (1).

**Table 1**  
*Budgetary efficiency of travellers (%)*

	<i>efficient</i>	<i>inefficient</i>	<i>total</i>
full tariff	31.9	26.6	58.5
reduced tariff	31.9	9.6	41.5
total	63.8	36.2	100.0

Table 1 reports the frequencies of “efficient” and “inefficient” travellers, where an “efficient traveller” is defined as an individual who buys the right type of train ticket, given his demand for train kilometres. Formally, a traveller who owns a reduction ticket and has reported travelling  $k$  kilometres is called efficient if

$$c_f(k) \geq c_r(k), \quad (6)$$

where  $c_f(\cdot)$  is the full tariff cost function and  $c_r(\cdot)$  is the reduced tariff cost function.<sup>7</sup> A similar definition can be given for “full tariff efficiency”. Table 1 shows that nearly half the full tariff travellers would gain by buying a reduced fare card. In the case of reduced fare travellers it still appears that about 23 per cent of the travellers travel too little to make the purchase of the reduced fare card a good investment. Note that travellers who are categorised as “inefficient” at this point are precisely those

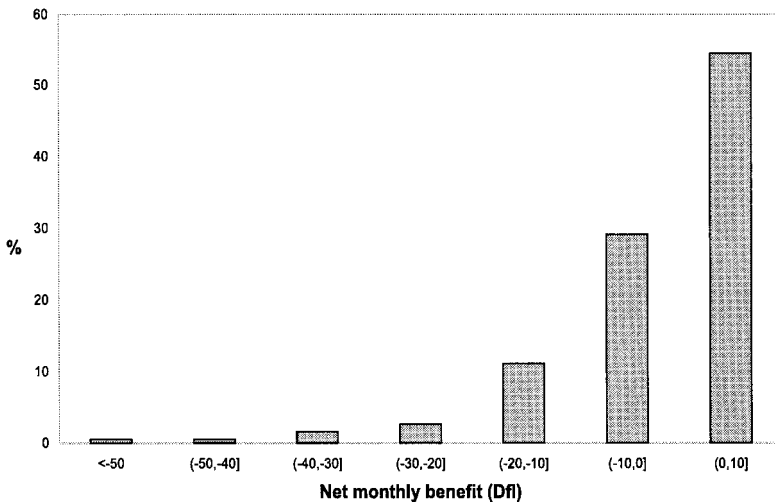
<sup>7</sup>Note that in the model of Section 2 this inequality is equivalent with (4).

travellers who locate themselves at a suboptimal part of the budget set, that is, the inner line DBE. A more detailed picture of the distribution of the net benefits is given in Figures 3 and 4. These figures depict an absolute measure of efficiency, which is defined as the difference between the cost of the ticket type that is not chosen and the cost of the chosen ticket type. (For reduction card holders this benefit equals  $c_f(k) - c_r(k)$ .) On average, one is indifferent between the two types of tickets at a yearly consumption of 1240 train kilometres. The many negative values in Figure 3, some with considerable magnitude, as compared to the few negative values in Figure 4, all with low magnitude, suggest that many individuals are reluctant to buy a reduced fare ticket. Note that both the maximum benefit of a full tariff traveller and the maximum loss of a reduced tariff traveller equal the price of a reduced fare card, which can be clearly seen in both histograms. The price of a reduced fare card equalled Dfl 9 per month (Dfl 108 per year) in 1992 and Dfl 8.25 per month (Dfl 99 per year) in 1993.

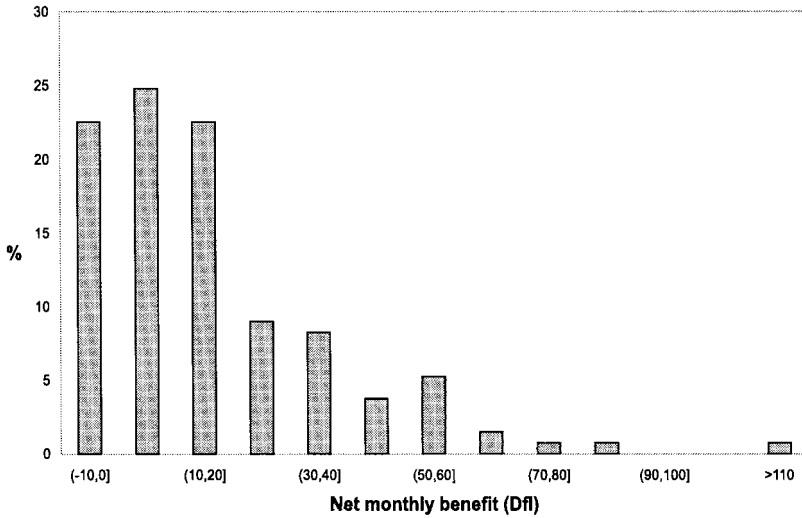
It should be noted that the current findings bear much resemblance to Moffitt (1983)'s ascertained disutility from participation in a welfare programme, and the finding of Koning and Ridder (1997) that there is an "entry fee effect" for households that might apply for rent assistance. For the current case, several explanations are possible for the discrepancy between what is observed and what might have been expected on grounds of a basic microeconomic specification. The most important explanation is probably that travellers have to make their choice of ticket type in advance, which means that the decision involves a certain risk. In prin-

**Figure 3**

*Monthly net benefits of full tariff travellers, compared to the "reduced tariff" option*



**Figure 4**  
*Monthly net benefits of reduced tariff travellers, compared to the "full tariff" option*



ciple, one buys a reduced fare card for the period of one year, so the person who invests Dfl 108 to buy a reduced fare card takes a risk because he is not certain about his future travel behaviour. This theory is in line with the findings of Train, Ben-Akiva and Atherton (1989), who find that consumers who are able to "self-select" among different telephone tariffs "do not choose tariffs with complete knowledge of their demand, but rather choose tariffs at least partly on the insurance provided by the tariff in the face of uncertain consumption patterns." Other possible explanations are that travellers are not well informed about the different types of travel cards, and that there exist some application costs for the purchase of a reduction card. As examples of the latter, one may think of waiting time at the ticket-window, and time and effort to make a photo — which is required in the Netherlands — and to fill in an application form.

## Econometric Specification

### Inclusion of take-up cost for reduction cards

The previous section has shown that many travellers experience a certain threshold before they buy a reduced fare card. Indeed there are also some reduced tariff travellers who appear to be on the wrong side of their budget

constraint, but these are easily outnumbered by the “inefficient” full tariff travellers. In order to take this into account in the economic model of Section 2, a fixed *take-up cost*  $T$  can be incorporated into the decision process that has been described in (3) and (4). The latter equation then modifies into:

$$\begin{aligned} v(hp_f, Y - p_r - T) &> v(p_f, Y) \Leftrightarrow \\ p_r + T &< \gamma Y. \end{aligned} \quad (7)$$

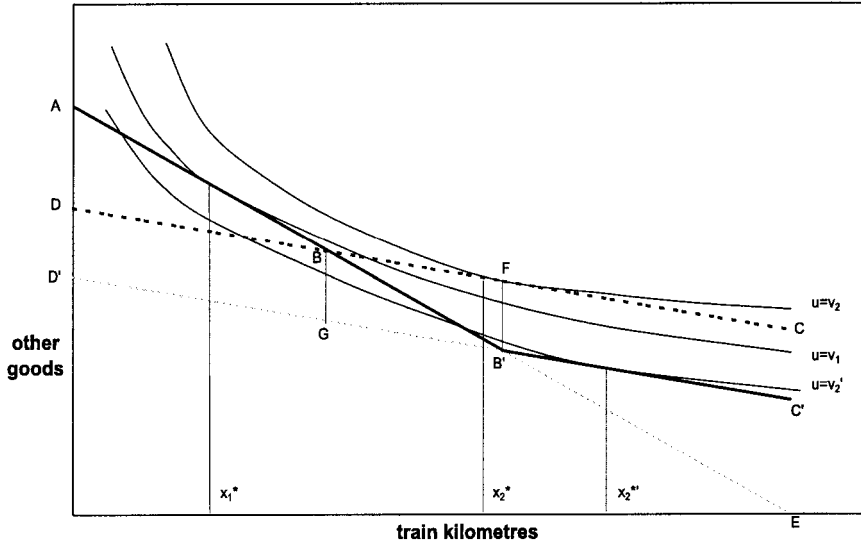
The expression for  $\gamma$  remains unaltered, as in (5). It is seen that if  $T > 0$ , which is expected on the grounds of Section 3, the customer is more reluctant to buy a reduced fare card than would have been expected on grounds of the theory in Section 2. In Figure 5 the implications for the microeconomic model are shown: with the introduction of a take-up cost  $DD'$ , the budget constraint changes to  $AB'C'$ , the conditional demand for the reduced fare ticket changes to  $x_2^{**}$ , and the indifference curve through this conditional demand is drawn by the line  $u = v_2'$ . Clearly, this figure shows that the take-up cost of a reduced fare card can make this card less attractive than the option of full tariff tickets. Whereas the original conditional demand  $x_2^*$  was superior to the conditional demand for the full tariff option, the existence of take-up costs means that the full tariff option is superior to the purchase of a reduction card in this example. Thus the optimal demand now equals  $x_1^*$ , due to its superiority over the conditional demand with a reduced fare card  $x_2^{**}$ . Figure 5 shows that individuals who locate at  $BB'$  now locate on their budget frontier, which implies that they are behaving perfectly efficiently. The other side of the coin is that the consumers in the segment  $BF$ , who are efficient with zero take-up costs, are shifted to the segment  $GB'$  in the case of positive take-up costs, implying that they are no longer efficient. A comparison of Figures 3 and 4 suggests that the number of travellers on  $BF$  is smaller than on  $BB'$ , so that the introduction of positive take-up costs leads to a larger number of efficient travellers.

Finally note that this new feature of the model does not impose any new restrictions. If the original model without take-up costs is valid, this will simply follow from the estimations by not being able to reject the null hypothesis that “take-up cost equals zero”. If this null hypothesis is rejected, however, there will be proof that there is a take-up cost involved for people who consider buying a reduced fare card.

### Individual tastes

In the model that has been described in the second section it was assumed that the parameters  $\alpha$ ,  $\beta$  and  $\delta$  are fixed. It is likely, however, that different individuals will have different tastes, and therefore different parameter values.

**Figure 5**  
Positive take-up cost implies reluctance to buy a reduced fare card



In general two ways exist to allow for such variation in taste: first, one can make one or more of the parameters depend on individual characteristics that are likely to influence the individuals' taste; second, one may specify a stochastic distribution for one or more of the parameters in order to account for unobserved taste variation. This latter extension is discussed in the next subsection. In this paper it is assumed that *nonstochastic preference heterogeneity* can be captured in the constant term  $\delta_i$  for individual  $i$  as follows:

$$\delta_i = S_i' \delta \quad (8)$$

where  $S_i$  is a vector with individual characteristics. As possible elements of  $S_i$  one may think of age, car possession, and other factors of interest. Note that, to allow for a constant term that is not individual-specific, the first element of  $S_i$  equals 1 for all individuals.

### Stochastic specification

The crucial step in going from a theoretical economic model towards an empirical model is the stochastic specification. Hausman (1985) gives an excellent overview of the various sources of randomness in simultaneous discrete/continuous choice models and suggests various specifications to incorporate these sources into an econometric model. The two most

interesting phenomena that are to be modelled as stochastic error terms are *preference heterogeneity* and *optimisation error*. As previously mentioned, apart from observed characteristics that may have influence on one's taste, it is likely that there is some *unobserved* heterogeneity in the preferences of travellers. The general way to specify an error term that takes account of this is to let one parameter vary according to a stochastic specification. The parameters of this stochastic specification are then to be estimated alongside the other parameters of the model. Optimisation error occurs because consumers do not always have the opportunity to purchase the desired amount of train kilometres. For example, someone may fall ill, which means that he is not able to buy his optimal consumption package. An additional source of optimisation error is caused by the fact that travellers are able to choose their own type of train ticket. As has already been argued, this can be partly modelled with an additional (deterministic) constant, which is a part of what was termed "take-up cost" above. On the other hand it is likely that there are some unobserved factors that may cause someone to choose an "inefficient" combination of ticket type and train kilometres. These can then be modelled by a stochastic error term. Third there is *specification error*, which is caused by the modeller's inability to specify a perfect econometric model. The usual way to incorporate such an error term is by introducing an additive error term in the demand function. This error term does however not appear in the indirect utility function, as its mere existence is caused by the modeller and the traveller does not have anything to do with it while making his optimal choice of a ticket type. The last form of randomness that is mentioned by Hausman is *measurement error*, which is caused by bad measurement of the data. In the present context, the translation from yearly data to monthly data constitutes an important part of this measurement error. Travel data are on a monthly basis, while the decision to purchase a reduction card or not is in principle taken on a yearly basis. The general problem of measurement error is, however, a whole issue in itself in the econometric literature, and will therefore not be considered in detail in this paper. A useful summary can be found in Greene (1993).

One can imagine that with all these sources of stochasticity it is possible to construct a wide variety of econometric specifications that go together with (1). In this paper the following two-error model will be estimated:

$$\ln k = \alpha \ln y + \beta \ln p + \delta + \varepsilon, \quad \varepsilon \sim N(0, \sigma^2) \quad (9a)$$

$$v(p, y) = \frac{1}{1 - \alpha} y^{1 - \alpha} - \frac{1}{1 + \beta} p^{1 + \beta} e^{\delta + v}, \quad v \sim N(0, \tau^2). \quad (9b)$$

In this specification the two error terms  $\varepsilon$  and  $v$  are assumed to be independently distributed. Specification error is taken account of by the stochastic term  $\varepsilon$ , while the error term  $v$  is characterised by the above described optimisation error. The current stochastic specification does not include a separate error term to incorporate stochastic preference heterogeneity, but this is a choice one has to make as it would not be possible to identify a third additional error term.

Having introduced stochastic terms into the model in (9), one is now able to derive the likelihood of each observation  $(k, r)$ , where  $k$  denotes the number of kilometres and  $r$  equals 1 if the individual owns a reduced fare card and 0 otherwise. As has been shown in (4) an individual chooses the reduced fare card option if (and only if) the indirect utility function for this option is higher than for the alternative; that is, travelling at the full rate. Substituting (9b) in (4) now yields

$$\delta + v > G, \quad (10)$$

where the function  $G \equiv G(Y, p_r, h, p_f)$  is given in Appendix A. The presence of the error term  $v$  allows one to write the probability that the individual owns a reduced fare card as

$$\begin{aligned} \Pr\{r = 1\} &= \Pr\{\delta + v > G\} = 1 - \Pr\{v < G - \delta\} \\ &= 1 - \Phi\left(\frac{G - \delta}{\tau}\right), \end{aligned}$$

and hence  $\Pr\{r = 0\} = \Phi\left(\frac{G - \delta}{\tau}\right)$ . The likelihood of owning a reduced fare card and travelling  $k$  kilometres can now be written as

$$\ell(k, r = 1) = f(k|r = 1) \cdot \Pr\{r = 1\},$$

where the density function  $f$  is given by

$$\begin{aligned} f(k|r = 1) &= f_\varepsilon(\ln k - \alpha \ln(Y - p_r) - \beta \ln(hp_f) - \delta) \\ &= \frac{1}{\sigma} \phi\left(\frac{\ln k - \alpha \ln(Y - p_r) - \beta \ln(hp_f) - \delta}{\sigma}\right). \end{aligned}$$

This density function  $f(k|r = 1)$  gives the probability density for travelling  $k$  kilometres, given that one owns a reduced fare card. Similarly, the likelihood of travelling  $k$  kilometres with full tariff tickets is expressed in terms of the conditional density function of travelling  $k$  kilometres and the probability of not owning a reduced fare pass:

$$\ell(k, r = 0) = f(k|r = 0) \cdot \Pr\{r = 0\},$$



where

$$f(k|r=0) = \frac{1}{\sigma} \phi \left( \frac{\ln k - \alpha \ln Y - \beta \ln p_f - \delta}{\sigma} \right).$$

Summing over all the individuals  $i$  of the data set, the total log-likelihood becomes

$$\ln \ell = \sum_{i:r=0} \ln \ell_i(k, r=0) + \sum_{i:r=1} \ln \ell_i(k, r=1).$$

This function is to be maximised to give the maximum likelihood estimates of the model in (9). Results will be reported in the next section.

## Estimation

### Design

The estimations focus on persons who had to make a choice between different types of tickets. This means that groups that receive free travel cards have been excluded. In the Netherlands this mainly concerns students, people from the army, and railway personnel (around 12 per cent of the sample). Only Second Class passengers (about 90 per cent of the sample) have been selected in the final data set.

The fixed cost of a reduced fare card ( $p_r$ ) equalled Dfl 9 in 1992 and Dfl 8.25 in 1993 (on a monthly basis). In the estimations the full tariff kilometre price has been fixed at the sample average kilometre price of Dfl 0.22. In the Netherlands, the level of the marginal tariff per kilometre depends on the length of a trip. It starts with a rather high value of around Dfl 0.30 for short trips and slightly decreases to around Dfl 0.10 for longer trips. For very long trips, the marginal price can even equal zero. In general, one would be able to make use of two different values for the price of a train kilometre: First, the average *individual* kilometre price ( $\bar{p}_i$ ) may be used in the microeconomic model, and second, the average kilometre price of the whole population ( $\bar{p}$ ) can be used. Thus, the first method allows for individually differentiated prices, while the second makes use of a “uniform” price. An advantage of the first method is that this “measurement” of price will be more precise at the individual level. However, it should be noted that  $\bar{p}_i$  is in general not equal to the marginal price of a train kilometre, which is of course the price that one is interested in.<sup>8</sup> In

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<sup>8</sup>To see this, consider the nonlinear budget constraint  $p(k) + H = y$ , with the differentiable kilometre expense function  $p(\cdot)$ . The Lagrange function equals  $\mathcal{Q} = u(k, H) - \lambda(p(k) + H - y)$ , and hence the first order condition for utility maximisation is:  $u_k/u_H = p'(k)$ . Thus, it is this marginal price  $p'(k)$  that we are interested in, and not the average price  $p(k)/k$ .

that respect it remains uncertain, *a priori*, which method is better. The main reason that in this paper  $\bar{p}$  has been used instead of  $\bar{p}_i$ , is that the latter method constitutes a potential source for endogeneity bias. In this sense,  $\bar{p}$  seems to be the best available instrument for the marginal price of each individual, as (a) it safeguards the estimates from endogeneity bias, while  $\bar{p}_i$  does not, and (b) it is not necessarily less precise than the  $\bar{p}_i$  method because both methods involve average prices, and eventually it is the marginal price that one is interested in.

As the original data set did not contain information on the respondents' income, their income has been replaced by a proxy variable which has been constructed according to the model in Appendix B. Although this inclusion of "predicted income" into the model may bring about some measurement error, it is to be preferred over negligence of this essential explanatory variable.<sup>9</sup> Note that under the assumption of independence between the error term of the income regression in Appendix B and the error term in the demand equation (9a), this procedure is essentially equivalent to the use of instrumental variables.

The vector  $S_i$  in (8), which allows for taste heterogeneity of individuals, contains variables on age, car availability, household size, and a dummy that indicates whether the individual is a housewife or not. The exact expression for  $\delta$  is as follows:

$$\delta = \delta_0 + \delta_1 AGE1 + \delta_2 AGE2 + \delta_3 CAR1 + \delta_4 CAR2 + \delta_5 HHSIZE + \delta_6 HWIFE.$$

In this specification "AGE1" and "AGE2" are dummies indicating whether the individual is younger than 30 years of age, or at least 50 years of age, respectively. The dummy variables "CAR1" and "CAR2" signify "can always make use of a car" and "can often make use of a car" respectively.

## Results

Table 2 contains the results of the maximum likelihood estimation procedure that has been outlined in Section 4. Apart from the dummy variable for housewives, all parameter estimates turn out to be significant at a 5 per cent confidence level. Moreover, all the estimates have the sign that is consistent with prior expectations and with microeconomic theory. The results indicate that the demand for train kilometres is fairly inelastic with respect to income. With values of  $-1.24$  and  $-1.37$  the demand for train kilometres turns out to be price elastic. The model with take-up costs turns out to be superior to the model without these costs, as can be read from

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<sup>9</sup> See Greene (1993).

**Table 2**  
*Model estimates, n = 6579*

<i>log-likelihood</i>		–14004		–13807	
$\sigma$		1.0463		1.0283	
$\tau$		0.6966		0.4608	
Parameters		Estimates	Std.err.	Estimates	Std.err.
$\delta$ :	constant	1.6619	0.2221	1.8981	0.1751
	age < 30	–0.0879	0.0216	–0.0743	0.0166
	age $\geq$ 50	0.1630	0.0231	0.1252	0.0179
	car available (always)	–0.2170	0.0216	–0.1675	0.0171
	car available (often)	–0.0993	0.0272	–0.0808	0.0208
	household size	–0.0766	0.0077	–0.0600	0.0062
	housewife	–0.0343	0.0230	–0.0183	0.0173
ln income		0.1445	0.0285	0.0988	0.0211
ln price		–1.2386	0.0501	–1.3732	0.0514
take-up cost		—	—	3.0606	0.1790

the likelihood scores and the low standard error of the take-up cost parameter. Moreover, the number of inefficient travellers (in the sense of (6)) decreased by 7.3 per cent with the introduction of this threshold parameter.

It can be seen from the table that the estimate of the price elasticity in the model without take-up costs is lower (in an absolute sense) than the price elasticity in the model with take-up costs. Thus negligence of the take-up cost effect leads to under-estimation of the price sensitivity of travellers. In other words, in the model without take-up costs the perceived reluctance of travellers to buy a reduced fare card is solely explained from their relative insensitivity for price differences, which clearly leads to under-estimation of the price elasticity. The model with take-up cost explains the reluctance to buy a reduced fare card not only from relative price insensitivity, but also from other factors mentioned earlier. Hence the first conclusion is that, given the high significance of the take-up parameter, reluctance to buy a reduced fare card need not only be attributed to the relative price insensitivity of a group of individuals. Motives such as risk aversion and inadequate information induce a threshold effect that is independent of “marginal price effects”. Comparing the absolute value of this threshold effect, Dfl 36.73 on a yearly basis,<sup>10</sup> with the price of a reduction card, Dfl 108 in 1992, and Dfl 99 in 1993, it

<sup>10</sup> Results in Table 2 are on a *monthly* basis.

can be seen that the average take-up cost equals 35 per cent of the actual price of a reduction card. The second conclusion is that the estimate for the price elasticity equals  $-1.37$  and thus that the off-peak demand for train kilometres is quite sensitive with regard to price.

The estimation implies that people who are younger than 30 years of age ( $-7.2$  per cent), people with high ( $-15.4$  per cent) or medium ( $-7.8$  per cent) car availability, and housewives ( $-1.8$  per cent) have less demand for train kilometres than others. On the other hand, people who are older than 50 years of age on average demand 13.3 per cent more train kilometres than others. The negative effect of the household size variable is most probably due to the fact that families with children prefer to travel by car, as the additional costs are zero in this case.

As mentioned at the end of Section 2, the reported estimates should satisfy the *Slutsky condition* in order to constitute a valid microeconomic model. Fortunately, the restriction was satisfied for *all* respondents for all different estimations.

### **Interpretation of the estimated price elasticity**

As shown in Section 3, the current study is based on a cross-section data set. This means that exactly one observation is available for each individual who has participated in the underlying survey. The static nature of this data set implies that the current microeconomic analysis aims to describe an equilibrium status, and thus that the reported price elasticities should be interpreted as *long-run* estimates of the *average* price sensitivity of travellers by train. It is likely that short-run effects of price changes are smaller in magnitude, in particular because reduced fare card holders have committed themselves to a certain way of travelling during a given period. In other words, substitution effects for reduction card holders will often be “lagged” until the expiration date of the reduction card.

In the past, several attempts have been made to estimate price elasticities for the demand for transport by train. Oum, Waters II and Yong (1992) give an overview of price elasticities for various means of transport, mainly based on studies from the 1980s. There appears to be some dispersion between estimates, but this can mainly be attributed to the type of data that has been used (cross-section, panel, aggregate time-series) as well as to the market segment (business, leisure) for which the elasticity has been estimated. As mentioned by Oum (1989) comparison of different estimates is often difficult, especially when the models to be compared are not nested in a more general model. Nevertheless, if the data source and the model assumptions are well taken into account, comparison of different elasticity estimates can be fruitful. Based on an aggregate time-series

data set, Oum (1992) reports a price elasticity of  $-1.16$  for the Netherlands (1990). This study, however, does not distinguish between peak and off-peak hour demand, nor between First and Second Class travellers. Given the widely adopted notion that peak hour demand is less elastic than the off-peak demand for train kilometres, the off-peak price elasticity of  $-1.37$  that has been estimated in this paper perfectly fits this figure.

### **Welfare implications of a positive take-up cost**

Clearly, this study shows that the purchase of a reduced fare card involves a large take-up cost. We briefly discuss the welfare implications of this.

As far as lack of information is the cause of the existence of take-up costs, there may be room for a Pareto improvement by providing better information to travellers. In any case this will make all travellers at least as well off as they were. Better information means that individuals have a better perception of their budget constraints, which *de facto* implies a rise in income. In Figure 5 this translates into a rise of the segment B'C', and thus will always yield at least as good a consumption package as the traveller already had. For the railway company, the effect of such an information measure is ambiguous. Better information not only affects people who were already locating *at* their budget constraint, but may also induce "irrational" travellers to improve their utility by moving towards a consumption package for which their budget constraint is binding. For instance, the railway company can often make an extra profit out of people who travel at full tariff while they would have been better off if they travelled at the reduced tariff (that is, users on B'E in Figure 5). Better information will surely make the number of these unwise travellers fall. The conclusion is that there exist both negative and positive effects on the profits of the railway company, and that the overall effect remains unclear *a priori*. Thus the overall welfare effect remains undetermined, and will in general depend on the extent to which better information is provided. The inability to foresee one's future behaviour has essentially the same welfare implications as the information factor. The only difference is that one's ability to foresee future behaviour cannot easily be improved, and thus the practical realisation of a possible welfare improvement is difficult for this case.

Second, the existence of take-up costs is an important factor in the eventual design of a modified tariff structure. Whereas standard welfare analysis provides clear-cut results on how to increase total welfare, or even on how to obtain a Pareto-improvement (see Brown and Sibley (1986)), the present case does not — in general — allow for an application of these results. This implies that each proposed modification of the tariff structure

has to be carefully examined, thereby taking into account the take-up costs that are involved. See Train *et al.* (1989) for a more elaborate discussion.

## Conclusion and Discussion

In this paper a model for the demand for train kilometres has been proposed and estimated. The approach that was used has been utilised previously in other areas of interest, such as labour supply and electricity demand, but has hardly received attention in the area of travel demand. As there appear to be many people who demand a combination of ticket type and train kilometres that is not optimal, this study has paid special attention to this issue by introducing a “take-up cost” parameter into the model.

The current estimates show that the price elasticity equals  $-1.37$  and that the income elasticity is fairly small at about  $0.1$ . Inclusion of a take-up cost in the model clearly shows that on average individuals are more reluctant to buy a reduced fare pass than would otherwise have been expected. To be precise, the average take-up cost equals Dfl 36.72 on a yearly basis, which equals 35 per cent of the price of a reduction card. There are several possible explanations for this large take-up cost. First, people who want to buy a reduction card have to take action to purchase it. Such action may include waiting time, filling in forms, and so on. Another possibility is that people are not well aware of their opportunities. In other words, they lack information about the different types of train tickets. A final explanation is that people who consider buying a reduction card have to make the decision in advance. People who are uncertain about their future travel behaviour may therefore choose to “wait and see” instead of taking the pre-emptive action of buying a reduced fare card.

To conclude this paper some possible extensions of the model are mentioned, which may be used in future analyses. First, as mentioned above, it is possible to construct a wide variety of econometric specifications that have the economic model of Section 2 as a baseline.

One of the most interesting extensions would be to specify the price elasticity as a random variable. Such a *random coefficient specification*, which has been used by Burtless and Hausman (1978) among others, would be of particular interest because the optimal pricing policy of a natural monopolist typically hinges on the fact that consumers have different price elasticities. (See Varian (1989), in particular the section on “Second-degree price discrimination”, for an excellent account.) Identification of the *spread* of price elasticities would then allow for constructing

the optimal tariff design, which is of course very interesting for both the railway company and the government.

Second, as it is likely that there is individual variation in take-up costs, an appealing extension of the current model would be to let the take-up cost parameter vary with each individual. This variation can be achieved through the inclusion of individual-specific parameters in the take-up cost specification, analogous to the specification in (8), or by making the take-up cost parameter stochastic, or even both.

Finally, an interesting extension of this study would be to let the current demand model be part of a larger demand system. Such a model could, for example, include the demand for train kilometres during peak hours. An interesting feature of such more integrated models would be the determination of substitution effects with other types of demands. Substitution effects may exist between First Class/Second Class demand, the afore mentioned peak/off-peak hour demand, and possibly also between other means of transport. The inclusion of zero-observations is also an interesting extension, as the behaviour of non-travellers may be quite relevant for both policy makers and the Netherlands Railways (NS).

## Appendix A Ticket Choice

In this appendix we give a functional form for  $G$ , such that the equivalence in (10) holds for all parameter values and all price and income levels:

$$G(Y, p_r, h, p_f) := (1 - \alpha) \ln Y - (1 + \beta) \ln p_f - \ln |1 - \alpha| + \ln |1 + \beta| + \\ \ln \left| 1 - \left( 1 - \frac{p_r}{Y} - \frac{T}{Y} \right)^{1-\alpha} \right| - \ln |1 - h^{1+\beta}|.$$

This equation is derived by rearranging (4), taking logs and rearranging once more. The absolute signs appear in order to allow for possibly income- and/or price-elastic demand, that is,  $\alpha > 1$  and/or  $\beta < -1$  respectively.

## Appendix B The Income Equation

Making use of the “OSA-arbeidsaanbodpanel 1985–88” the following income equation has been estimated:

$$\ln INCOME = 5.72 + 0.542 \ln EDU + 0.231 \ln AGE - 0.633I\{FEM\}$$

(0.172)    (0.0401)            (0.0399)            (0.0256)

$$R^2 = 0.325$$

In this model the logarithm of income has been specified as a linear function of (the logarithmic transformations of) education (*EDU*), age (*AGE*) and gender (*FEM*). This last variable equals 1 if the individual is a female, and 0 otherwise. Income is defined as the net personal monthly income. Standard errors are denoted in parentheses below the corresponding estimates. All parameters turn out to be highly significant at the 5 per cent and 1 per cent level. It is seen that “gender” explains a large share of the variation in individual income: on average the model predicts that women earn 47 per cent less than men. Of course “discrimination” is not the only explanation for this gap, as in the Netherlands labour participation of women is lower and many women who do work have part time jobs. More sophisticated specifications have not been taken into consideration since an important side constraint is that the explanatory variables to be included have to be present in both the OSA and the NS data sets.

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